IFT-P.044/99 gr-qc/9905096

Search for semiclassical–gravity effects in relativistic stars

Daniel A.T. Vanzella* and George E.A. Matsas†

Instituto de Física Teórica, Universidade Estadual Paulista,

Rua Pamplona 145, 01405-900, São Paulo, SP

Brazil

Abstract

We discuss the possible influence of gravity in the neutronization process, $p^+ \; e^- \; \to n \; \nu_e$, which is particularly important as a cooling mechanism of neutron stars. Our approach is semiclassical in the sense that leptonic fields are quantized on a classical background spacetime, while neutrons and protons are treated as excited and unexcited nucleon states, respectively. We expect gravity to have some influence wherever the energy content carried by the in–state is barely above the neutron mass. In this case the emitted neutrinos would be soft enough to have a wavelength of the same order as the space curvature radius.

04.62.+v, 04.40.Dg

Typeset using REVTEX

*vanzella@ift.unesp.br

†matsas@ift.unesp.br

The inner structure of neutron stars has attracted much attention of relativists and particle and nuclear physicists since there still remain many subtle points to be better understood (see, e.g., [1] and references therein). It would be interesting, thus, to investigate how gravity could influence quantum processes which occur in their interior. The gravitational field may be of some significance to quantum phenomena wherever they involve particles with wavelengths of the same order as the space curvature radius. Processes dealing with soft particles are very promising since their wavelengths may be arbitrarily large. Here we focus on the neutronization process, $p^+ \, e^- \to n \, \nu_e$, which is an important cooling mechanism for neutron stars with temperatures up to about 10^9 K. Our approach is essentially semiclassical in the sense that leptonic fields are quantized on a classical background spacetime, while neutrons and protons are treated as excited and unexcited nucleon states, respectively. We will use natural units $G = \hbar = c = k_B = 1$ throughout this paper.

Field quantization in the Schwarzschild spacetime is not easy to accomplish [2]. We shall simplify the problem by simulating the Schwarzschild spacetime by a two–dimensional noninertial frame described by the Rindler wedge. The Rindler wedge is a static spacetime defined by the line element

$$ds^2 = a^2 u^2 d\tau^2 - du^2 \tag{1}$$

with $0 < u < +\infty$ and $-\infty < \tau < +\infty$, where a "characterizes" the frame acceleration, i.e., $a \equiv \sqrt{a_\mu a^\mu} = \text{const}$ is the proper acceleration of the worldline which has τ as its proper time, namely, $u = a^{-1}$.

We would like to consider the case in which the nucleons lie approximately static during the reaction at some fixed point in the star. In principle, this poses no problem since the whole process takes place in the presence of a medium and not in the vacuum. The location where the reaction happens will be specified by the nucleon proper acceleration. Thus, in our simplified model the reacting nucleons will be described by a uniformly accelerated current in the Rindler wedge with constant proper acceleration a:

$$j^\mu = q u^\mu \delta(u - a^{-1}), \tag{2}$$

where q is a small coupling constant and $u^\mu = (a, 0)$ is the nucleon four–velocity. Next, in order to allow the current above to describe the proton–neutron transition, we shall consider the nucleon as a two–level system [3]. In this vein, neutrons $|n\rangle$ and protons $|p\rangle$ will be excited and unexcited eigenstates of the nucleon Hamiltonian $\hat{H}$:

$$\hat{H}|n\rangle = m_n|n\rangle \;\; , \;\; \hat{H}|p\rangle = m_p|p\rangle \;\; , \tag{3}$$

where m_n and m_p are the neutron and proton masses, respectively. Hence current (2) will be replaced by

$$\hat{j}^\mu = \hat{q}(\tau) u^\mu \delta(u - a^{-1}) \;\; , \tag{4}$$

where $\hat{q}(\tau) \equiv \exp(i\hat{H}\tau) \; \hat{q}_0 \; \exp(-i\hat{H}\tau)$ is a Hermitian monopole. The two–dimensional Fermi constant $G_F \equiv |\langle p|\hat{q}_0|n\rangle| = 9.918 \times 10^{-13}$ is determined [4] by imposing that the mean proper lifetime of inertial neutrons is 887 s [5].

In order to calculate the neutronization rate we shall quantize the leptonic field in the Rindler wedge. The leptonic field is expressed as [6]

$$\hat{\Psi}(\tau,u) = \sum_{\sigma=\pm} \int_0^{+\infty} d\omega \left(\hat{b}_{\omega\sigma}\psi_{\omega\sigma}(\tau,u) + \hat{d}^{\dagger}_{\omega\sigma}\psi_{-\omega-\sigma}(\tau,u) \right), \tag{5}$$

where $\psi_{\omega\sigma}(\tau,u) = f_{\omega\sigma}(u)e^{-i\omega\tau}$ are positive ($\omega > 0$) and negative ($\omega < 0$) frequency solutions of the Dirac equation, with respect to the boost Killing field $\partial/\partial\tau$, with polarizations $\sigma = \pm$. We recall that Rindler frequencies may assume arbitrary positive real values. In particular there are massive Rindler particles with arbitrarily small frequencies. (See Ref. [7] for a discussion on zero-frequency Rindler particles.) Here

$$f_{\omega+}(u) = A_+ \begin{pmatrix} K_{i\omega/a+1/2}(mu) + iK_{i\omega/a-1/2}(mu) \\ 0 \\ -K_{i\omega/a+1/2}(mu) + iK_{i\omega/a-1/2}(mu) \\ 0 \end{pmatrix}, \tag{6}$$

$$f_{\omega-}(u) = A_- \begin{pmatrix} 0 \\ K_{i\omega/a+1/2}(mu) + iK_{i\omega/a-1/2}(mu) \\ 0 \\ K_{i\omega/a+1/2}(mu) - iK_{i\omega/a-1/2}(mu) \end{pmatrix}, \tag{7}$$

where m is the lepton mass and the normalization constants

$$A_+ = A_- = \left[\frac{m\cosh(\pi\omega/a)}{2\pi^2 a} \right]^{1/2} \tag{8}$$

were chosen such that the annihilation and creation operators satisfy the following simple anticommutation relations

$$\{\hat{b}_{\omega\sigma}, \hat{b}^{\dagger}_{\omega'\sigma'}\} = \{\hat{d}_{\omega\sigma}, \hat{d}^{\dagger}_{\omega'\sigma'}\} = \delta(\omega-\omega')\,\delta_{\sigma\sigma'}, \tag{9}$$

$$\{\hat{b}_{\omega\sigma}, \hat{b}_{\omega'\sigma'}\} = \{\hat{d}_{\omega\sigma}, \hat{d}_{\omega'\sigma'}\} = \{\hat{b}_{\omega\sigma}, \hat{d}_{\omega'\sigma'}\} = \{\hat{b}_{\omega\sigma}, \hat{d}^{\dagger}_{\omega'\sigma'}\} = 0 \;. \tag{10}$$

Now we are ready to calculate the neutronization amplitude

$$\mathcal{A} = \langle n| \otimes \langle \nu_{\omega_\nu\sigma_\nu}|\; \hat{S}_I\; |e^-_{\omega_e\sigma_e}\rangle \otimes |p\rangle, \tag{11}$$

where we minimally couple the nucleon current (4) to the leptonic fields $\hat{\Psi}_e$ and $\hat{\Psi}_\nu$ through the Fermi interaction action

$$\hat{S}_I = \int d^2x\sqrt{-g}\; \hat{j}_\mu(\hat{\bar{\Psi}}_\nu\gamma^\mu_R\hat{\Psi}_e + \hat{\bar{\Psi}}_e\gamma^\mu_R\hat{\Psi}_\nu)\;. \tag{12}$$

In the Rindler wedge $\gamma^\mu_R \equiv (e_\alpha)^\mu\gamma^\alpha$ with tetrads $(e_0)^\mu = u^{-1}\delta^\mu_0$ and $(e_i)^\mu = \delta^\mu_i$, where γ^α are the usual Dirac matrices. By using Eq. (12) in Eq. (11), we obtain the following amplitude:

$$\mathcal{A}_{ac} = G_F \int_{-\infty}^{+\infty} d\tau\; e^{i\Delta m\tau}\langle \nu_{\omega_\nu\sigma_\nu}|\hat{\Psi}^{\dagger}_\nu(\tau,a^{-1})\hat{\Psi}_e(\tau,a^{-1})|e^-_{\omega_e\sigma_e}\rangle, \tag{13}$$

where $\Delta m \equiv m_n - m_p$. Next, by using Eq. (5), we obtain

$$\mathcal{A}_{ac} = G_F\,\delta_{\sigma_e,\sigma_\nu} \int_{-\infty}^{+\infty} d\tau\; e^{i\Delta m\tau}\psi^{\dagger}_{\omega_\nu\sigma_\nu}(\tau,a^{-1})\;\psi_{\omega_e\sigma_e}(\tau,a^{-1})\;. \tag{14}$$

Using now explicitly $\psi_{\omega\sigma}(\tau, u)$ to perform the integral, we obtain

$$\mathcal{A}_{ac} = \frac{4G_F}{\pi a}\sqrt{m_e m_\nu \cosh(\pi\omega_e/a)\cosh(\pi\omega_\nu/a)} \\ \times Re\left[K_{i\omega_\nu/a-1/2}(m_\nu/a)\ K_{i\omega_e/a+1/2}(m_e/a)\right]\delta_{\sigma_e,\sigma_\nu}\delta(\omega_e-\omega_\nu-\Delta m)\ . \tag{15}$$

This result will be used to calculate the total reaction rate

$$\Gamma_{ac}(a) \equiv \frac{1}{\tilde{\tau}}\sum_{\sigma_e=\pm}\sum_{\sigma_\nu=\pm}\int_0^{+\infty} d\omega_e \int_0^{+\infty} d\omega_\nu |\mathcal{A}_{ac}|^2 n_F(\omega_e, T_e)[1-n_F(\omega_\nu, T_\nu)]\ , \tag{16}$$

where $\tilde{\tau} = 2\pi\delta(0)$ is the total nucleon proper time [8], and $n_F(\omega, T) \equiv 1/[1+exp(\omega/T)]$ is the usual fermionic thermal factor. We shall consider further two cases. In the first one, we assume $T_e = 10^9$ K and $T_\nu = 0$ K, i.e., the neutron star would be cold enough to be transparent to the neutrinos. In the second one, we assume $T_e = T_\nu = 10^{10}$ K, i.e., electrons and neutrinos would be in thermal equilibrium. By using Eq. (15) in Eq. (16), we obtain

$$\Gamma_{ac}(a) = \frac{4G_F^2 m_e m_\nu}{\pi^3 a^2}\int_{\Delta m}^{+\infty} d\omega_e \frac{\cosh[\pi\omega_e/a]\cosh[\pi(\omega_e-\Delta m)/a]\exp[(\omega_e-\Delta m)/2T_\nu]}{\cosh[\omega_e/2T_e]\cosh[(\omega_e-\Delta m)/2T_\nu]\exp[\omega_e/2T_e]} \\ \times \left\{Re\left[K_{i(\omega_e-\Delta m)/a-1/2}(m_\nu/a)K_{i\omega_e+1/2}(m_e/a)\right]\right\}^2\ . \tag{17}$$

As a final step, we take the limit $m_\nu \to 0$ in Eq. (17) (see Ref. [9]):

$$\Gamma_{ac}(a) = \frac{G_F^2 m_e}{\pi^2 a}\int_{\Delta m}^{+\infty} d\omega_e \frac{\cosh[\pi\omega_e/a]\exp[(\omega_e-\Delta m)/2T_\nu]}{\cosh[\omega_e/2T_e]\cosh[(\omega_e-\Delta m)/2T_\nu]\exp[\omega_e/2T_e]} \\ \times K_{i\omega_e/a+1/2}(m_e/a)K_{i\omega_e/a-1/2}(m_e/a)\ . \tag{18}$$

In order to compare the reaction rate above with the usual one obtained in inertial frames, we calculate next the reaction rate for $a = 0$ using plain quantum field theory in Minkowski spacetime. This will be used also as a consistency check since we will compare it with the $a \to 0$ limit obtained from Eq. (18).

Let us briefly outline the Minkowski calculation. The leptonic fields will be expressed in terms of the usual Minkowski coordinates (t, z) as

$$\hat{\Psi}(t,z) = \sum_{\sigma=\pm}\int_{-\infty}^{+\infty} dk\left(\hat{b}_{k\sigma}\psi_{k\sigma}^{(+\omega)}(t,z) + \hat{d}_{k\sigma}^\dagger\psi_{-k-\sigma}^{(-\omega)}(t,z)\right)\ , \tag{19}$$

where $\hat{b}_{k\sigma}$ and $\hat{d}_{k\sigma}^\dagger$ are annihilation and creation operators of fermions and antifermions, respectively, with momentum k and polarization σ. In the inertial frame, energy, momentum and mass m are related as usual: $\omega = \sqrt{k^2+m^2} > 0$. $\psi_{k\sigma}^{(+\omega)}(t,z)$ and $\psi_{k\sigma}^{(-\omega)}(t,z)$ are positive and negative frequency solutions of the Dirac equation with respect to $\partial/\partial t$, respectively. In the Dirac representation (see, e.g., Ref. [8]), we find

$$\psi_{k+}^{(\pm\omega)}(t,z) = \frac{e^{i(\mp\omega t+kz)}}{\sqrt{2\pi}}\begin{pmatrix}\pm\sqrt{(\omega\pm m)/2\omega}\\ 0\\ k/\sqrt{2\omega(\omega\pm m)}\\ 0\end{pmatrix} \tag{20}$$

and

$$\psi_{k-}^{(\pm\omega)}(t,z) = \frac{e^{i(\mp\omega t + kz)}}{\sqrt{2\pi}} \begin{pmatrix} 0 \\ \pm\sqrt{(\omega \pm m)/2\omega} \\ 0 \\ -k/\sqrt{2\omega(\omega \pm m)} \end{pmatrix} , \tag{21}$$

where the normalization constants were chosen such that the creation and annihilation operators satisfy

$$\{\hat{b}_{k\sigma}, \hat{b}^{\dagger}_{k'\sigma'}\} = \{\hat{d}_{k\sigma}, \hat{d}^{\dagger}_{k'\sigma'}\} = \delta(k-k')\;\delta_{\sigma\sigma'} \tag{22}$$

and

$$\{\hat{b}_{k\sigma}, \hat{b}_{k'\sigma'}\} = \{\hat{d}_{k\sigma}, \hat{d}_{k'\sigma'}\} = \{\hat{b}_{k\sigma}, \hat{d}_{k'\sigma'}\} = \{\hat{b}_{k\sigma}, \hat{d}^{\dagger}_{k'\sigma'}\} = 0 \;\; . \tag{23}$$

The neutronization amplitude for inertial nucleons in the Minkowski spacetime,

$$\mathcal{A}_{in} = G_F \int_{-\infty}^{+\infty} dt\; e^{i\Delta m t} \psi_{k_\nu\sigma_\nu}^{(+\omega_\nu)\dagger}(t,0)\; \psi_{k_e\sigma_e}^{(+\omega_e)}(t,0) \;, \tag{24}$$

is calculated by using the interaction action (12) in Eq. (11), where γ^μ_R is replaced by the usual γ^μ Dirac matrices, and the current is given by $\hat{j}^\mu = \hat{q}(t) v^\mu \delta(z)$ with $v^\mu = (1,0)$. This leads us straightforwardly to the following neutronization rate for inertial nucleons:

$$\Gamma_{in} = \frac{2G_F^2}{\pi} \int_L^{+\infty} dk_e \frac{e^{(\omega_e - \Delta m)/T_\nu}}{(1+e^{\omega_e/T_e})[1+e^{(\omega_e-\Delta m)/T_\nu}]} \;, \tag{25}$$

where $m_\nu = 0$, $L \equiv \sqrt{\Delta m^2 - m_e^2}$, and we recall that $\omega_e \equiv \sqrt{k_e^2 + m_e^2}$.

In order to clearly analyze the influence of the frame acceleration on the neutronization process, let us use Eqs. (18) and (25) to define the following relative reaction rate:

$$\mathcal{R}(a) \equiv \frac{\Gamma_{ac}(a) - \Gamma_{in}}{\Gamma_{in}} \;. \tag{26}$$

In Figs. 1 and 2 we plot $\mathcal{R}(a)$ for the two aforementioned cases: (i) $T_e = 10^9$ K and $T_\nu = 0$ K, and (ii) $T_e = T_\nu = 10^{10}$ K. Firstly we note from the figures that Γ_{ac} $(a \to 0)$ is in agreement with the expression obtained for Γ_{in} since $\mathcal{R}(a \to 0) \to 0$. Figs. 1 and 2 exhibit a complicated oscillatory pattern up to $a \approx 1$ MeV. Indeed, the frame acceleration plays its most important role in this region: $|\mathcal{R}(a)|$ reaches about 30% and 10% for cases (i) and (ii), respectively. For large enough accelerations, $a \gg \Delta m,\ T_e$, we obtain from Eq. (18) an asymptotic expression for Γ_{ac}, namely,

$$\Gamma_{ac}(a \gg \Delta m,\ T_e) \approx \frac{2G_F^2}{\pi} \int_{\Delta m}^{+\infty} d\omega_e \frac{e^{(\omega_e - \Delta m)/T_\nu}}{(1+e^{\omega_e/T_e})[1+e^{(\omega_e-\Delta m)/T_\nu}]} \;. \tag{27}$$

By using Eq. (27) in Eq. (26), we can compute the asymptotic relative reaction rate, namely, $\mathcal{R}(a \gg \Delta m,\ T_e)$. We find that $\mathcal{R}(a \gg \Delta m,\ T_e) \approx -7.2\%$ and $\mathcal{R}(a \gg \Delta m,\ T_e) \approx -3.5\%$

for cases (i) and (ii), respectively, i.e., according to our toy model, ultrahigh accelerations damp the neutronization rate by a few percents.

In summary, we have looked for gravity effects in the neutronization process which frequently occurs in the interior of neutron stars. The reaction rate obtained by means of a simplified model exhibits a complicated oscillatory pattern up to $a \approx 1$ MeV. Afterwards it tends to an asymptotic value which indicates that the reaction is somewhat damped. We note that proper accelerations of the order $a \approx 1$ MeV are much beyond what would be expected in the interior of relativistic stars. Just for sake of comparison, protons at LHC/CERN will be under accelerations of about 10^{-8} MeV. We emphasize, however, that only a four–dimensional Schwarzschild calculation would be realistic enough to precisely determine the whole influence of gravity in the neutronization reaction and other similar processes. In a more realistic calculation, for instance, effects due to the *space curvature* itself, which is absent here, should show up wherever the emitted neutrinos are soft enough to "feel" the global background geometry. In this case, even reactions taking place at the star core, where $a \approx 0$, would be influenced by gravity. More detailed investigations on the role played by gravity in particle processes occuring in relativistic stars would be welcome.

Acknowledgments

D.V. was fully supported by Fundação de Amparo à Pesquisa do Estado de São Paulo while G.M. was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico.

REFERENCES

[1] S. Bonazzola, E. Gourgoulhon and J. Marck, Phys. Rev. Lett. **82**, 892 (1999); P.B. Jones, Phys. Rev. Lett. **81**, 4560 (1998); J. Madsen, Phys. Rev. Lett. **81**, 3311 (1998); P.M. Pizzochero, L. Viverit and R.A. Broglia, Phys. Rev. Lett. **79**, 3347 (1997); N.K. Glendenning, S. Pei and F. Weber, Phys. Rev. Lett. **79**, 1603 (1997); A.G. Wiseman, Phys. Rev. Lett. **79**, 1189 (1997); P.R. Brady and S.A. Hughes, Phys. Rev. Lett. **79**, 1186 (1997); P.B. Jones, Phys. Rev. Lett. **79**, 792 (1997).

[2] B.P. Jensen and P. Candelas, Phys. Rev. D **33**, 1590 (1986); Erratum *ibid.* D **35**, 4041 (1987).

[3] W.G. Unruh, Phys. Rev. D **14**, 870 (1976); B.S. DeWitt, *General Relativity* , eds. S.W. Hawking and W. Israel (Cambridge University Press, Cambridge, 1979).

[4] G.E.A. Matsas and D.A.T. Vanzella, Phys. Rev. D **59**, 094004 (1999).

[5] C. Caso et al., The European Phys. Journ. **C3**, 1 (1998).

[6] P. Candelas and D. Deutsch, Proc. R. Soc. Lond. A. **362**, 251 (1978); W. Troost and H. Van Dam, Nucl. Phys. **B152**, 442 (1979); M. Soffel, B. Muller and W. Greiner, Phys. Rev. D **22**, 1935 (1980); R. Járegui, M. Torres and S. Hacyan, Phys. Rev. D **43** 3979 (1991); E. Bautista, Phys. Rev. D **48** 783 (1993).

[7] A. Higuchi, G.E.A. Matsas and D. Sudarsky, Phys. Rev. D **45**, R3308 (1992); ibid **46**, 3450 (1992).

[8] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).

[9] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series and Products* (Academic Press, New York, 1980).

Figure Captions

FIG. 1: The relative reaction rate $\mathcal{R}(a)$ is plotted as a function of the frame acceleration a for temperatures $T_e = 10^9$ K and $T_\nu = 0$ K. Note that $\mathcal{R}(a \to 0) \to 0$, as expected. After an oscillatory regime the relative reaction rate tends to the asymptotic value $\mathcal{R}(a \gg \Delta m,\ T_e) \approx -7.2\%$. The maximum value reached by $|\mathcal{R}(a)|$ is about 30%.

FIG. 2: The relative reaction rate $\mathcal{R}(a)$ is plotted as a function of the frame acceleration a for temperatures $T_e = T_\nu = 10^{10}$ K. After an oscillatory regime the relative reaction rate tends to the asymptotic value $\mathcal{R}(a \gg \Delta m,\ T_e) \approx -3.5\%$. The maximum value reached by $|\mathcal{R}(a)|$ is about 10%.

FIGURES

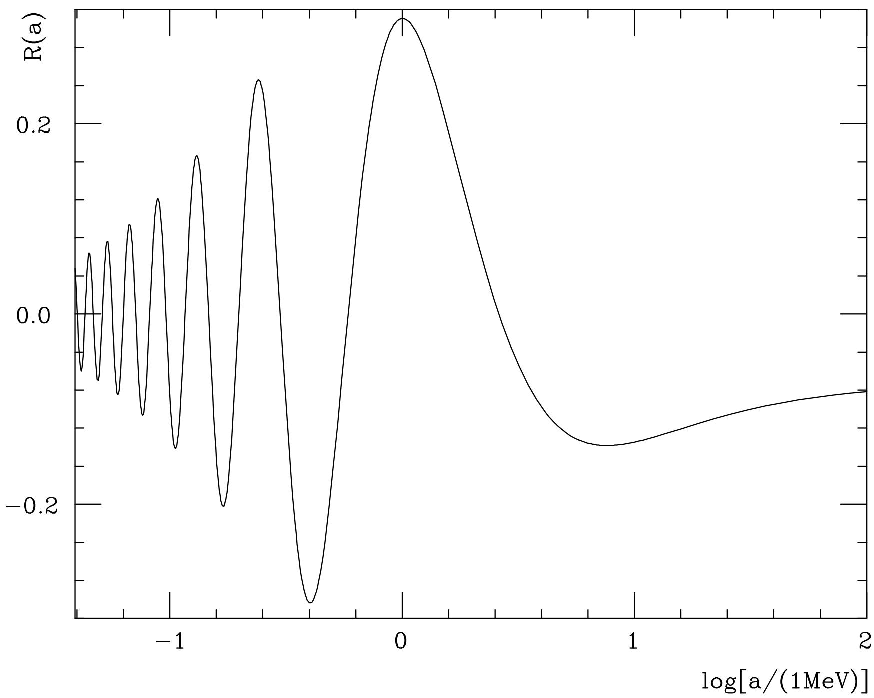

FIG. 1.

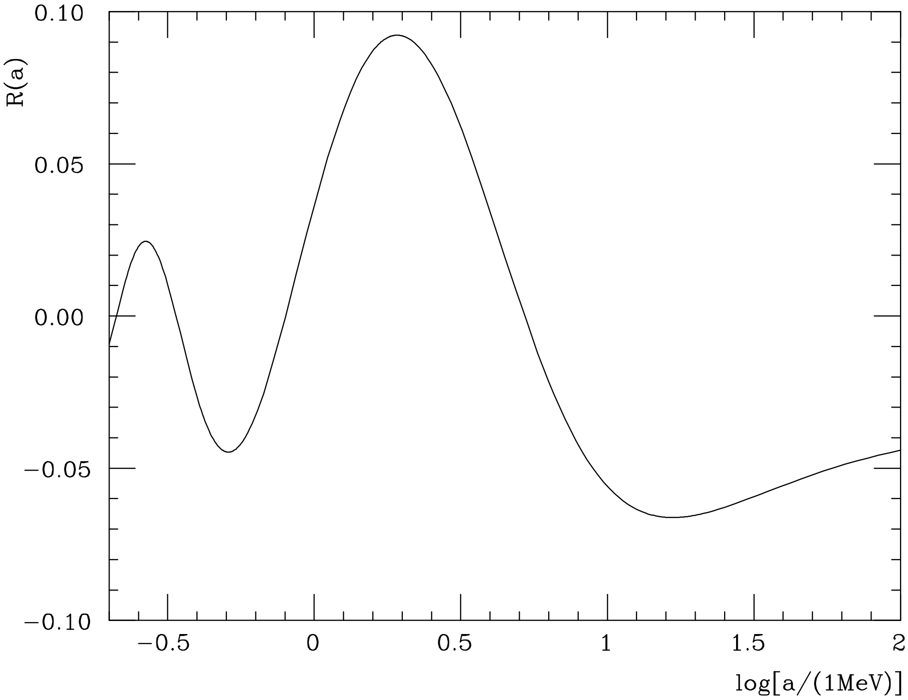

FIG. 2.